## LIGHT POLARIZING BY MAGNETIC FIELDS

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#### Abstract

According to an ancient photonic model it is possible to light polarize without crystals. This amazing model evidences that the photon is not a boson but a "faithful wedded pair" of fermions with the same mass and opposite electric charge. Its intimate structure, with concordant angular momenta and discordant spins, gives rise to a paramagnetism which, in presence of a magnetic field, causes gyroscope motions with typical Larmor precessions, linked in the orientation of magnetic field and magnetic moment; if the magnetic field has a gradient it is possible to polarize the angular momenta states. The experimental evidence can be obtained using a complicated Stern Gerlach apparatus: although the high vacuum is no more necessary and the furnace is substituted by a humble light bulb, optical loops or traps (like buffer gas cells) are needed, in the gap of the electromagnet, in order to increase the extremely low cross time which the deflection depends on. The theoretic and practical consequences are considerable and all of them to be rediscovered.


## The Nala-Damayanti model

The Mahabharata clearly gives us an accurate structure of the photon in the legend of Nala and Damayanti ${ }^{1}$. According to this "innovative" model two particles (the most beautiful of the universe) with the same mass and opposite electric charge, Damayanti $\boldsymbol{\delta}$ (negative) and Nala $\boldsymbol{v}$ (positive), united in an unbreakable couple, oscillate, along the three directions of the space, around the same orbital " $S$ " in a hydrogen-like system (fig. 1).


Figure 1

[^0]The $\boldsymbol{v}$ particle has a leading role, because it is able to impose on $\boldsymbol{\delta}$ a revolution which is concordant with its spin and angular momentum. This model doesn't violate the Pauli principle, according to which the only possible spin configuration in such an orbital is $\uparrow \downarrow$. Thus, as a matter of fact, we are forced to assume a zero total spin ( $\mathbf{S}_{\text {tot }}=\mathbf{S}_{\boldsymbol{\delta}}+\mathbf{S}_{v}=0$ ), while the total angular momentum obviously is $\mathbf{L}_{\text {tot }}=\mathbf{L}_{v}+\mathbf{L}_{\delta}=2 \mathbf{L}_{v}\left(\mathbf{L}_{\text {tot }}=0\right.$ is impossible because it would cause a destructive collision $)$.
The photon state therefore is very peculiar; the addition of its two spins cannot produce the states

 states $|+-\rangle_{\mathbf{u}}$ or $|-+\rangle_{\mathbf{u}}$ : the only allowed angular momenta states are $|++\rangle_{\mathbf{u}}$ or $|--\rangle_{\mathbf{u}}$. Furthermore $\mathbf{L}_{\text {tot }}$ is always parallel to the spins $\mathbf{S}_{\boldsymbol{\delta}}$ and $\mathbf{S}_{\mathbf{v}}$ ! We can then accurately avoid the separate additions of orbital angular momenta and spins and also the related classical four-dimensional state space calculations or, better still, the global sixteen-dimensional one, because they are a little more complicated and time wasting, with the same results. If we want to investigate $\mathbf{L}_{\text {tot }}$ and the effect produced by $\mathbf{S}_{\boldsymbol{\delta}}$ and $\mathbf{S}_{\mathbf{v}}$ (although we know that, being $\mathbf{S}_{\text {tot }}$ null, its measurements with any observable give null results), all we need is a simple two-dimensional state space calculation; that will be better understood in the next sections.

## A strong 2D character

Let $\mathbf{J}$ be one among $\mathbf{L}_{\mathbf{v}}, \mathbf{L}_{\delta}, \mathbf{S}_{\mathbf{v}}$, or $\mathbf{S}_{\boldsymbol{\delta}}$. If $j(j+1) \hbar^{2}$ and $m \notin$ denote the eigenvalues of $\mathbf{J}^{2}$ and $J_{z}$, it is easily verifiable for photons that the only possible value for $j$ is the first positive half-integer $1 / 2$, while $m$ can take on the values $-j$ or $j$. We obtain the two-dimensional observables:

$$
\mathbf{J}^{2}=j(j+l) h^{2}\left(\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right)=\frac{3}{4} h^{2}\left(\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right) \quad J_{z}=m h\left(\begin{array}{rr}
1 & 0 \\
0 & -1
\end{array}\right)=\frac{h}{2}\left(\begin{array}{rr}
1 & 0 \\
0 & -1
\end{array}\right)
$$

This scheme applies to every single angular momentum and spin; thus we obtain:

$$
\begin{array}{ll}
\mathbf{L}_{v}{ }^{2}=\frac{3}{4} h^{2}\left(\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right) & L_{v z}=\frac{h}{2}\left(\begin{array}{rr}
1 & 0 \\
0 & -1
\end{array}\right) \\
\mathbf{L}_{\delta}{ }^{2}=\frac{3}{4} h^{2}\left(\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right) & L_{\delta \delta}=\frac{h}{2}\left(\begin{array}{rr}
1 & 0 \\
0 & -1
\end{array}\right) \\
\mathbf{S}_{v}{ }^{2}=\frac{3}{4} h^{2}\left(\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right) & S_{v z}=\frac{h}{2}\left(\begin{array}{rr}
1 & 0 \\
0 & -1
\end{array}\right) \\
\mathbf{S}_{\delta}{ }^{2}=\frac{3}{4} h^{2}\left(\begin{array}{ll}
1 & 0 \\
0 & 1
\end{array}\right) & S_{\delta \delta}=\frac{h}{2}\left(\begin{array}{rr}
1 & 0 \\
0 & -1
\end{array}\right)
\end{array}
$$

Angular momenta and spins have the same behaviour; they show a pronounced two-dimensional character: that's why we got the spin of the photon mixed up with its angular momentum!

## Larmor precession

It is clear that a photon, being neutral, is not subjected to the Lorentz force, but in a magnetic field something happens, because of the opposite electric charge of the two particles $\boldsymbol{v}$ and $\boldsymbol{\delta}$ : while the concordant angular momenta don't produce any effect (there is steadily a null resultant magnetic moment $\mathbf{M}_{\delta}+\mathbf{M}_{v}=0$ ), two discordant spins generate concordant magnetic moments; thus the permanent total magnetic moment $\mathscr{M}_{\text {tot }}=\mathscr{M}_{\delta}+\mathscr{M}_{v}=\mathscr{\mathscr { M }}_{v}$ (fig. 2) gives the photon an unexpected
paramagnetic behaviour. It is useful noting that the permanent moment $\mathscr{\mathscr { M }}_{\text {tot }}$ is the only measurable physical quantity which makes us trace back the spins $\mathbf{S}_{\boldsymbol{\delta}}$ and $\mathbf{S}_{\mathbf{v}}$.


Figure 2
Since $\mathscr{\mathscr { M }}_{v}=\gamma \mathbf{S}_{v}$ with $\gamma$ gyromagnetic ratio (positive for $v$ particle, unlike the electron), if we consider a photon in a uniform magnetic field $\mathbf{B}$ and choose the $z$ axis along $\mathbf{B}$ (fig. 3), the total moment $\Gamma_{\text {tot }}$ acting on the photon is the vector product:


Figure 3

$$
\Gamma_{\mathrm{tot}}=2 \mathscr{M}_{v} \times \mathbf{B}=\mathscr{M}_{\mathrm{tot}} \times \mathbf{B}
$$

but, since $\mathbf{S}_{\text {tot }}$ is constantly null with its time-derivatives, the angular momentum theorem states:

$$
\frac{d}{d t} \mathbf{L}_{\text {tot }}=\Gamma_{\text {tot }}
$$

that's to say:

$$
\frac{d}{d t} \mathbf{L}_{\text {tot }}=\mathscr{\mathscr { M }}_{\mathrm{tot}} \times \mathbf{B}=2 \mathscr{M}_{v} \times \mathbf{B}=2 \gamma \mathbf{S}_{v} \times \mathbf{B}=\gamma \mathbf{L}_{\mathrm{tot}} \times \mathbf{B}
$$

As $d \mathbf{L}_{\text {tot }} / \mathrm{dt}$ is perpendicular to $\mathbf{B}$ and $\mathbf{L}_{\text {tot }}$, performing a scalar multiplication on both members of this equation by either $\mathbf{L}_{\text {tot }}$ or $\mathbf{B}$, we obtain:

$$
\mathbf{L}_{\text {tot }} \cdot \frac{d}{d t} \mathbf{L}_{\text {tot }}=0 \quad \rightarrow \quad \frac{d}{d t}\left(\mathbf{L}_{\text {tot }}\right)^{2}=0
$$

$$
\mathbf{B} \cdot \frac{d}{d t} \mathbf{L}_{\mathrm{tot}}=0 \quad \rightarrow \quad \frac{d}{d t}\left(\mathbf{B} \cdot \mathbf{L}_{\mathrm{tot}}\right)=0
$$

The photon thus behaves like a gyroscope: the angular momentum $\mathbf{L}_{\text {tot }}$ evolves with a constant modulus, maintaining a constant angle $\vartheta$ with $\mathbf{B}$ and the rotational angular velocity about the $z$ axis depends on $\gamma$ and $\mathbf{B}$. We can deduce that the photon has a typical Larmor precession.
The classical potential energy is the scalar product:

$$
W=-2 \mathscr{M}_{v} \cdot \mathbf{B}=-2 \mathscr{M}_{v z} B=-2 \gamma B S_{v z}
$$

where $B$ is the modulus of the magnetic field and $S_{v z}$ is the component of $\mathbf{S}_{v}$ along $z$. This energy, expressed through $L_{\text {totz }}$, the component of $\mathbf{L}_{\text {tot }}$ along $z$, becomes:

$$
W=-\gamma B L_{\text {totz }}=\frac{\omega}{2} L_{\text {totz }}
$$

where $\omega=-2 \gamma B$ (negative because $\gamma$ is positive).

## Convenient state subspaces

The addition of orbital angular momenta and spins gives:

$$
\mathbf{J}_{\text {tot }}=\mathbf{L}_{v}+\mathbf{L}_{\delta}+\mathbf{S}_{v}+\mathbf{S}_{\delta}
$$

The resultant $\mathbf{J}_{\text {tot }}$ would require a sixteen-dimensional state space in the orthonormal base


 (eigenvalue $-h$ ) can be found in a measurement of the observable $J_{z}$, which have ten non-null elements only in its diagonal:

$$
J_{z}=\left(\begin{array}{cccccccccccccccc}
2 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & 1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & 1 & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & 0 & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & 1 & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & 0 & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & 0 & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & -1 & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & 1 & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & 0 & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & 0 & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & -1 & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & 0 & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & -1 & . & . \\
. & . & . & . & . & . & . & . & . & . & . & . & . & . & -1 & . \\
. & . & . & . & . & . & . & . & -2
\end{array}\right)
$$

that's to say we need only the two-dimensional subspace spanned by the eigenvectors $|+++-\rangle$ and $|---+\rangle$ ! Eliminating all the rows and columns, except for the second and second-last rows and for the second and second-last columns, $J_{z}$ simply becomes:

$$
J_{z}=h\left(\begin{array}{rr}
1 & 0 \\
0 & -1
\end{array}\right)
$$

with simplified eigenvectors $|+\rangle$ (corresponding to $|+++-\rangle$ ) and $|-\rangle$ (corresponding to
 angular momentum state $\mathbf{L}_{\text {tot }}$ has the observable $L_{z}$ :

$$
L_{z}=h\left(\begin{array}{rr}
1 & 0 \\
0 & -1
\end{array}\right)
$$

As $\mathscr{M}_{\text {tot }}$ behave like $\mathbf{L}_{\text {tot }}$ (fig. 2) we can directly have the observable $M_{z}$ of $\mathscr{\mathscr { M }}_{\text {tot }}$ in the bidimensional format:

$$
M_{z}=\gamma h\left(\begin{array}{rr}
1 & 0 \\
0 & -1
\end{array}\right)
$$

The reduction of the observable $J_{x}$ is more fascinating. $J_{x}$ is obtainable by summing $J_{x 1}, J_{x 2}, J_{x 3}$ and $J_{x 4}$ :

$$
J_{x}=\frac{h}{2}\left(\begin{array}{cccccccccccccccc}
. & 1 & 1 & . & 1 & . & . & . & 1 & . & . & . & . & . & . & . \\
\mathbf{1} & . & . & 1 & . & 1 & . & . & . & 1 & . & . & . & . & . & . \\
1 & . & . & 1 & . & . & 1 & . & . & . & 1 & . & . & . & . & . \\
. & 1 & 1 & . & . & . & . & 1 & . & . & . & 1 & . & . & . & . \\
1 & . & . & . & . & 1 & 1 & . & . & . & . & . & 1 & . & . & . \\
. & 1 & . & . & 1 & . & . & 1 & . & . & . & . & . & 1 & . & . \\
. & . & 1 & . & 1 & . & . & 1 & . & . & . & . & . & . & 1 & . \\
. & . & . & 1 & . & 1 & 1 & . & . & . & . & . & . & . & . & 1 \\
1 & . & . & . & . & . & . & . & . & 1 & 1 & . & 1 & . & . & . \\
. & 1 & . & . & . & . & . & . & 1 & . & . & 1 & . & 1 & . & . \\
. & . & 1 & . & . & . & . & . & 1 & . & . & 1 & . & . & 1 & . \\
. & . & . & 1 & . & . & . & . & . & 1 & 1 & . & . & . & . & 1 \\
. & . & . & . & 1 & . & . & . & 1 & . & . & . & . & 1 & 1 & . \\
. & . & . & . & . & 1 & . & . & . & 1 & . & . & 1 & . & . & 1 \\
. & . & . & . & . & . & 1 & . & . & . & 1 & . & 1 & . & . & 1 \\
. & . & . & . & . & . & . & 1 & . & . & . & 1 & . & 1 & 1 & .
\end{array}\right)
$$

The only possible eigenvectors $|+++-\rangle_{\mathrm{x}}$ and $|---+\rangle_{\mathrm{x}}$ define the two-dimensional subspace with the eigenvalues $\pm h$ for $J_{x}$; detailing their eigenbras (only for mere convenience) we have:

$$
\begin{aligned}
& { }_{x}<+++-\left\lvert\,=\frac{1}{4}\left(\begin{array}{llllllllllllllll}
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1
\end{array}\right)\right. \\
& { }_{x}<---+\left\lvert\,=\frac{1}{4}\left(\begin{array}{llllllllllllllll}
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 & -1 & -1
\end{array}\right)\right.
\end{aligned}
$$

Keeping only the first two rows and columns on and doubling their value (in order to have the same eigenvalues), preserving the symmetry-antisymmetry of the two eigenvectors, $J_{x}$ simply becomes:

$$
J_{x}=h\left(\begin{array}{ll}
0 & 1 \\
1 & 0
\end{array}\right)
$$

such an observable is characterized by the eigenvalues $\pm 4$ and the eigenvectors $1 / \sqrt{2}(|+\rangle+|-\rangle)$ (corresponding to $|+++-\rangle_{\mathrm{x}}$ ) and $1 / \sqrt{2}\left(|+\rangle-|-\rangle\right.$ ) (corresponding to $|---+\rangle_{\mathrm{x}}$ ). The $2 \times 2$ matrices $L_{x}$ and $M_{x}$ we are searching for are:

$$
L_{x}=h\left(\begin{array}{ll}
0 & 1 \\
1 & 0
\end{array}\right) \quad M_{x}=\gamma h\left(\begin{array}{ll}
0 & 1 \\
1 & 0
\end{array}\right)
$$

In an absolutely similar way we find out the observable $L_{y}$ and $M_{y}$ :

$$
L_{y}=h\left(\begin{array}{rr}
0 & -i \\
i & 0
\end{array}\right) \quad M_{y}=\gamma h\left(\begin{array}{rr}
0 & -i \\
i & 0
\end{array}\right)
$$

which have eigenvalues $\pm h$ and eigenvectors $1 / \sqrt{2}(|+\rangle \pm i|-\rangle)$ in a pure two-dimension state space.


Figure 4
The generic observables $L_{u}$ and $M_{u}$ are simple combinations among $L_{z}, L_{x}, L_{y}$ and $M_{z}, M_{x}, M_{y}$; while considering the polar angles $\vartheta$ and $\varphi$ (fig. 4) we have:

$$
\begin{aligned}
L_{u}=L_{z} \cos \vartheta+L_{x} \sin \vartheta \cos \varphi+L_{y} \sin \vartheta \sin \varphi & M_{u}=M_{z} \cos \vartheta+M_{x} \sin \vartheta \cos \varphi+M_{y} \sin \vartheta \sin \varphi \\
L_{u}=h\left(\begin{array}{ccc}
\cos \vartheta & \sin \vartheta e^{-i \varphi} \\
\sin \vartheta e^{i \varphi} & -\cos \vartheta
\end{array}\right) & M_{u}=\gamma h\left(\begin{array}{cc}
\cos \vartheta & \sin \vartheta e^{-i \varphi} \\
\sin \vartheta e^{i \varphi} & -\cos \vartheta
\end{array}\right)
\end{aligned}
$$

such matrices have two non-degenerate eigenvalues $+h$ and $-h$ with the respective eigenvectors:

$$
\begin{aligned}
& \cos \frac{\vartheta}{2} e^{-i \varphi / 2}|+\rangle+\sin \frac{\vartheta}{2} e^{i \varphi / 2}|-\rangle \\
& -\sin \frac{\vartheta}{2} e^{-i \varphi / 2}|+\rangle+\cos \frac{\vartheta}{2} e^{i \varphi / 2}|-\rangle
\end{aligned}
$$

Summing up the observables of $\mathbf{L}_{\text {tot }}$ are:

$$
L_{z}=h\left(\begin{array}{rr}
1 & 0 \\
0 & -1
\end{array}\right) \quad L_{x}=h\left(\begin{array}{ll}
0 & 1 \\
1 & 0
\end{array}\right) \quad L_{y}=h\left(\begin{array}{rr}
0 & -i \\
i & 0
\end{array}\right) \quad L_{u}=h\left(\begin{array}{cc}
\cos \vartheta & \sin \vartheta e^{-i \varphi} \\
\sin \vartheta e^{i \varphi} & -\cos \vartheta
\end{array}\right)
$$

while those of $\mathscr{\mathscr { M }}_{\text {tot }}$ are:

$$
M_{z}=\gamma h\left(\begin{array}{rr}
1 & 0 \\
0 & -1
\end{array}\right) \quad M_{x}=\gamma h\left(\begin{array}{ll}
0 & 1 \\
1 & 0
\end{array}\right) \quad M_{y}=\gamma h\left(\begin{array}{rr}
0 & -i \\
i & 0
\end{array}\right) \quad M_{u}=\gamma h\left(\begin{array}{cc}
\cos \vartheta & \sin \vartheta e^{-i \varphi} \\
\sin \vartheta e^{i \varphi} & -\cos \vartheta
\end{array}\right)
$$

That's to say they have different eigenvalues ( $\pm \neq$ and $\pm \gamma / 4$ ) with the same eigenvectors: $|+\rangle$ and

 The complete sets of commuting observables (C.S.C.O.) for the angular momentum $\mathbf{L}_{\text {tot }}$ are $\mathbf{L}^{2}$ and one among $L_{z}, L_{x}, L_{y}$ and $L_{u}$ (e.g. $\left\{\mathbf{L}^{2}, L_{z}\right\}$ ), while for the magnetic moment $\mathscr{\mathscr { H }}_{\text {tot }}$ are $\mathbf{M}^{2}$ and one among $M_{z}, M_{x}, M_{y}$ and $M_{u}$ (e.g. $\left\{\mathbf{M}^{2}, M_{z}\right\}$ ), where:

$$
\mathbf{L}^{2}=L_{x}{ }^{2}+L_{y}{ }^{2}+L_{z}{ }^{2}=3 h^{2} I \quad \text { and } \quad \mathbf{M}^{2}=M_{x}{ }^{2}+M_{y}{ }^{2}+M_{z}{ }^{2}=3(\gamma h)^{2} I
$$

with $I$ the identity matrix.

## Evolution in a uniform magnetic field

Let $\vartheta$ and $\varphi$ be the polar angles of the vectors $\mathbf{L}_{\text {tot }}$ and $\mathscr{\mathscr { M }}_{\text {tot }}$ (fig. 4), the total angular momentum and the total magnetic moment of the photon, along the unit vector $\mathbf{u}$; in the basis of the eigenvectors $|+\rangle$ and $|-\rangle$, characterizing the observable $L_{z}$ and $M_{z}$, the total angular momentum state $|\chi(t)\rangle_{\mathbf{L}}$ (fig. 3a) and the parallel associated magnetic moment state $|\psi(t)\rangle_{\mathbf{M}}$, at time $t=0$, are:

$$
\begin{gathered}
|\chi(0)\rangle_{\mathbf{L}}=\cos \frac{\vartheta}{2} e^{-i \varphi / 2}|+\rangle+\sin \frac{\vartheta}{2} e^{i \varphi / 2}|-\rangle \\
|\psi(0)\rangle_{\mathbf{M}}=\cos \frac{\vartheta}{2} e^{-i \varphi / 2}|+\rangle+\sin \frac{\vartheta}{2} e^{i \varphi / 2}|-\rangle
\end{gathered}
$$

(They coincide because $L_{z}$ and $M_{z}$ commute). We can quantize the potential energy $W=(\omega / 2) L_{\text {totz }}$ replacing $L_{\text {totz }}$ by the observable $L_{z}$ and $W$ by the Hamiltonian $H$ which describes the evolution of the total angular momentum in the field $\mathbf{B}$; thus we have:

$$
H=\frac{\omega}{2} L_{z}=\frac{\omega}{2} h\left(\begin{array}{rr}
1 & 0 \\
0 & -1
\end{array}\right)
$$

which is clearly time-independent; since $H$ and $L_{z}$ have the same eigenvectors:

$$
\begin{aligned}
& H|+\rangle=+\frac{\omega}{2} h|+\rangle \\
& H|-\rangle=-\frac{\omega}{2} h|-\rangle
\end{aligned}
$$

there are two energy levels $E^{\circ}=+\hbar \omega / 2$ and $E^{\circ \circ}=-\hbar \omega / 2$; their difference $\hbar \omega$ defines the Bohr frequency $f_{B}=\omega / 2 \pi=\left(E^{\circ}-E^{\circ \circ}\right) / h$.
The state vector $|\chi(t)\rangle_{\mathbf{L}}$, at a generic time $t$, can be written:

$$
|\chi(t)\rangle_{\mathbf{L}}=a(t)|+\rangle+b(t)|-\rangle
$$

so we have a simply to solve Schrödinger equation:

$$
i \hbar \frac{d}{d t}|\chi(t)\rangle_{\mathbf{L}}=H|\chi(t)\rangle_{\mathbf{L}} \text { that is }\left\{\begin{array} { l } 
{ i h \frac { d } { d t } a ( t ) = \frac { \omega h } { 2 } a ( t ) } \\
{ i h \frac { d } { d t } b ( t ) = - \frac { \omega h } { 2 } b ( t ) }
\end{array} \text { or } \left\{\begin{array}{l}
\frac{d}{d t} a(t)=-\frac{i \omega}{2} a(t) \\
\frac{d}{d t} b(t)=\frac{i \omega}{2} b(t)
\end{array}\right.\right.
$$

with the solutions:

$$
\left\{\begin{array} { l } 
{ a ( t ) = A e ^ { - i \omega t / 2 } } \\
{ b ( t ) = B e ^ { i \omega t / 2 } }
\end{array} \quad \text { with } \quad \left\{\begin{array}{l}
A=a(0)=\cos \frac{\vartheta}{2} e^{-i \varphi / 2} \\
B=b(0)=\sin \frac{\vartheta}{2} e^{i \varphi / 2}
\end{array}\right.\right.
$$

The state vector $|\chi(t)\rangle_{\mathbf{L}}$ becomes:

$$
|\chi(t)\rangle_{\mathbf{L}}=\cos \frac{\vartheta}{2} e^{-i \varphi / 2} e^{-i \omega t / 2}|+\rangle+\sin \frac{\vartheta}{2} e^{i \varphi / 2} e^{i \omega t / 2}|-\rangle
$$

or, better still:

$$
|\chi(t)\rangle_{\mathbf{L}}=\cos \frac{\vartheta}{2} e^{-i(\varphi+\omega t) / 2}|+\rangle+\sin \frac{\vartheta}{2} e^{i(\varphi+\omega t) / 2}|-\rangle
$$

The parallel associated state $|\psi(t)\rangle_{\mathbf{M}}$, at a generic time $t$, is similarly calculated:

$$
|\psi(t)\rangle_{\mathbf{M}}=\cos \frac{\vartheta}{2} e^{-i(\varphi+\omega t) / 2}|+\rangle+\sin \frac{\vartheta}{2} e^{i(\varphi+\omega t) / 2}|-\rangle
$$

Comparing the last two expressions at a generic time $t$ with the correlated ones at time $t=0$, we can observe that the direction along which the total angular momentum is $+h$ and the magnetic moment is $+\gamma$ (fig. 3a), with certainty, has the polar angles:

$$
\left\{\begin{aligned}
\vartheta(t) & =\vartheta \\
\varphi(t) & =\varphi+\omega t
\end{aligned}\right.
$$

which are typical of a Larmor precession.

## Probabilities and mean values

The probabilities of obtaining $+h$ in a measurement of the observable $L_{z}$ and $+\gamma h$ in a measurement of the observable $M_{z}$, are the same (because the eigenvectors are the same):

$$
\begin{aligned}
& \left.\left|\langle+\mid \chi(t)\rangle_{\mathbf{L}}\right|^{2}=\left\lvert\, \begin{array}{ll}
1 & 0
\end{array}\right.\right)\left.\binom{\cos \frac{\vartheta}{2} e^{-i(\varphi+\omega t) / 2}}{\sin \frac{\vartheta}{2} e^{i(\varphi+\omega t) / 2}}\right|^{2}=\cos ^{2} \frac{\vartheta}{2} \\
& \left.\left|\langle+\mid \psi(t)\rangle_{\mathrm{M}}\right|^{2}=\left\lvert\, \begin{array}{ll}
1 & 0
\end{array}\right.\right)\left.\binom{\cos \frac{\vartheta}{2} e^{-i(\varphi+\omega t) / 2}}{\sin \frac{\vartheta}{2} e^{i(\varphi+\omega t) / 2}}\right|^{2}=\cos ^{2} \frac{\vartheta}{2}
\end{aligned}
$$

with $\omega$ depending on the gyromagnetic ratio and the magnetic field. Since the modulus of $e^{ \pm i(\varphi+\omega t) / 2}$ is unitary, these probabilities are time-independent. Analogously the probabilities of obtaining $-\psi$ in a measurement of the observable $L_{z}$ and $-\gamma h$ in a measurement of the observable $M_{z}$, are:

$$
\begin{aligned}
& \left.\left|\langle-\mid \chi(t)\rangle_{\mathrm{L}}\right|^{2}=\left\lvert\, \begin{array}{ll}
0 & 1
\end{array}\right.\right)\left.\binom{\cos \frac{\vartheta}{2} e^{-i(\varphi+\omega t) / 2}}{\sin \frac{\vartheta}{2} e^{i(\varphi+\omega t) / 2}}\right|^{2}=\sin ^{2} \frac{\vartheta}{2} \\
& \left.\left.\langle-\mid \psi(t)\rangle_{\mathrm{M}}\right|^{2}=\left\lvert\, \begin{array}{ll}
0 & 1
\end{array}\right.\right)\left.\binom{\cos \frac{\vartheta}{2} e^{-i(\varphi+\omega t) / 2}}{\sin \frac{\vartheta}{2} e^{i(\varphi+\omega t) / 2}}\right|^{2}=\sin ^{2} \frac{\vartheta}{2}
\end{aligned}
$$

time-independent too, while the mean value of $L_{z}$, for $\mathbf{L}_{\text {tot }}$ and the mean value of $M_{z}$ for $\mathscr{\mathscr { M }}_{\text {tot }}$ are also time-independent:

$$
\begin{aligned}
& { }_{\mathbf{L}}\langle\chi(t)| L_{z}|\chi(t)\rangle_{\mathbf{L}}=\left(\begin{array}{ll}
\cos \frac{\vartheta}{2} e^{i(\varphi+\omega t) / 2} & \sin \frac{\vartheta}{2} e^{-i(\varphi+\omega t) / 2}
\end{array}\right) h\left(\begin{array}{rr}
1 & 0 \\
0 & -1
\end{array}\right)\binom{\cos \frac{\vartheta}{2} e^{-i(\varphi+\omega t) / 2}}{\sin \frac{\vartheta}{2} e^{i(\varphi+\omega t) / 2}}= \\
& =h\left(\cos ^{2} \frac{\vartheta}{2}-\sin ^{2} \frac{\vartheta}{2}\right)=h \cos \vartheta \\
& { }_{\mathrm{M}}\langle\psi(t)| M_{\mathrm{z}}|\psi(t)\rangle_{\mathrm{M}}=\left(\begin{array}{ll}
\cos \frac{\vartheta}{2} e^{i(\varphi+\omega t) / 2} & \left.\sin \frac{\vartheta}{2} e^{-i(\varphi+\omega t) / 2}\right) \gamma h\left(\begin{array}{rr}
1 & 0 \\
0 & -1
\end{array}\right)\binom{\cos \frac{\vartheta}{2} e^{-i(\varphi+\omega t) / 2}}{\sin \frac{\vartheta}{2} e^{i(\varphi+\omega t) / 2}}=.=0 \cdot=~
\end{array}\right. \\
& =\gamma h\left(\cos ^{2} \frac{\vartheta}{2}-\sin ^{2} \frac{\vartheta}{2}\right)=\gamma h \cos \vartheta
\end{aligned}
$$

as we can see it is enough a simple arrangement ( $\gamma \boldsymbol{h}$ instead of $h$ ) for obtaining the mean value $\left\langle M_{z}\right\rangle$ from $\left\langle L_{z}\right\rangle$. Thus, also considering that the probabilities are the same while calculated for $\mathbf{L}_{\text {tot }}$ or $\mathscr{H}_{\text {tot }}$, from now on we shall neglect the redundant calculations.
The probabilities of obtaining $+h$ in a measurement of the observable $L_{x}$ and $+\gamma h$ in a measurement of the observable $M_{x}$, are:

$$
\begin{gathered}
\left|\mathbf{x}\langle+\mid \chi(t)\rangle_{\mathbf{L}}\right|^{2}=\left\lvert\,\left(\begin{array}{ll}
\frac{1}{\sqrt{2}} & \left.\frac{1}{\sqrt{2}}\right)\left.\binom{\cos \frac{\vartheta}{2} e^{-i(\varphi+\omega t) / 2}}{\sin \frac{\vartheta}{2} e^{i(\varphi+\omega t) / 2}}\right|^{2}= \\
=\left|\frac{1}{\sqrt{2}} \cos \frac{\vartheta}{2}\left(\cos \frac{\varphi+\omega t}{2}-i \sin \frac{\varphi+\omega t}{2}\right)+\frac{1}{\sqrt{2}} \sin \frac{\vartheta}{2}\left(\cos \frac{\varphi+\omega t}{2}+i \sin \frac{\varphi+\omega t}{2}\right)\right|^{2}= \\
=\left|\frac{1}{\sqrt{2}} \cos \frac{\vartheta}{2} \cos \frac{\varphi+\omega t}{2}+\frac{1}{\sqrt{2}} \sin \frac{\vartheta}{2} \cos \frac{\varphi+\omega t}{2}+\frac{i}{\sqrt{2}} \sin \frac{\vartheta}{2} \sin \frac{\varphi+\omega t}{2}-\frac{i}{\sqrt{2}} \cos \frac{\vartheta}{2} \sin \frac{\varphi+\omega t}{2}\right|^{2}= \\
=\left|\frac{1}{\sqrt{2}} \cos \frac{\varphi+\omega t}{2}\left(\cos \frac{\vartheta}{2}+\sin \frac{\vartheta}{2}\right)+\frac{i}{\sqrt{2}} \sin \frac{\varphi+\omega t}{2}\left(\sin \frac{\vartheta}{2}-\cos \frac{\vartheta}{2}\right)\right|^{2}= \\
=\frac{1}{2} \cos ^{2} \frac{\varphi+\omega t}{2}\left(\cos \frac{\vartheta}{2}+\sin \frac{\vartheta}{2}\right)^{2}+\frac{1}{2} \sin ^{2} \frac{\varphi+\omega t}{2}\left(\sin \frac{\vartheta}{2}-\cos \frac{\vartheta}{2}\right)^{2}= \\
=\frac{1}{2} \cos ^{2} \frac{\varphi+\omega t}{2}(1+\sin \vartheta)+\frac{1}{2} \sin ^{2} \frac{\varphi+\omega t}{2}(1-\sin \vartheta)= \\
=\frac{1}{2}+\frac{1}{2} \sin \vartheta\left(\cos ^{2} \frac{\varphi+\omega t}{2}-\sin ^{2} \frac{\varphi+\omega t}{2}\right)= \\
=\frac{1}{2}+\frac{1}{2} \sin \vartheta \cos (\varphi+\omega t)
\end{array},\right.\right.
\end{gathered}
$$

Analogously the probabilities of obtaining $-\psi$ in a measurement of the observable $L_{x}$ and $-\gamma h$ in a measurement of the observable $M_{x}$ are:

$$
\left|{ } _ { \mathbf { x } } \left(-\left.|\chi(t)\rangle_{\mathbf{L}}\right|^{2}=\frac{1}{2}-\frac{1}{2} \sin \vartheta \cos (\varphi+\omega t)\right.\right.
$$

The matrices $L_{x}$ and $M_{x}$, which like $L_{y}$ and $M_{y}$ don't commute with $H$, have time-dependent mean values:

$$
\begin{gathered}
{ }_{\mathbf{L}}\langle\chi(t)| L_{x}|\chi(t)\rangle_{\mathbf{L}}=\left(\begin{array}{ll}
\cos \frac{\vartheta}{2} e^{i(\varphi+\omega t) / 2} & \sin \frac{\vartheta}{2} e^{-i(\varphi+\omega t) / 2}
\end{array}\right) h\left(\begin{array}{ll}
0 & 1 \\
1 & 0
\end{array}\right)\binom{\cos \frac{\vartheta}{2} e^{-i(\varphi+\omega t) / 2}}{\sin \frac{\vartheta}{2} e^{i(\varphi+\omega t) / 2}}= \\
=h \sin \frac{\vartheta}{2} \cos \frac{\vartheta}{2}\left(e^{i(\varphi+\omega t)}+e^{-i(\varphi+\omega t)}\right)=h \sin \vartheta \cos (\varphi+\omega t) \\
{ }_{\mathbf{M}}\langle\psi(t)| M_{x}|\psi(t)\rangle_{\mathbf{M}}=\gamma h \sin \vartheta \cos (\varphi+\omega t)
\end{gathered}
$$

The probabilities of obtaining $+h$ in a measurement of the observable $L_{y}$ and $+\gamma h$ in a measurement of the observable $M_{y}$, are:

$$
\begin{aligned}
& \left|\mathbf{y}\langle+\mid \chi(t)\rangle_{\mathbf{L}}\right|^{2}=\left|\left(\begin{array}{ll}
\frac{1}{\sqrt{2}} & \frac{-i}{\sqrt{2}}
\end{array}\right)\binom{\cos \frac{\vartheta}{2} e^{-i(\varphi+\omega t) / 2}}{\sin \frac{\vartheta}{2} e^{i(\varphi+\omega t) / 2}}\right|^{2}= \\
& =\left|\frac{1}{\sqrt{2}} \cos \frac{\vartheta}{2}\left(\cos \frac{\varphi+\omega t}{2}-i \sin \frac{\varphi+\omega t}{2}\right)-\frac{i}{\sqrt{2}} \sin \frac{\vartheta}{2}\left(\cos \frac{\varphi+\omega t}{2}+i \sin \frac{\varphi+\omega t}{2}\right)\right|^{2}= \\
& =\left|\frac{1}{\sqrt{2}} \cos \frac{\vartheta}{2} \cos \frac{\varphi+\omega t}{2}+\frac{1}{\sqrt{2}} \sin \frac{\vartheta}{2} \sin \frac{\varphi+\omega t}{2}-\frac{i}{\sqrt{2}} \sin \frac{\vartheta}{2} \cos \frac{\varphi+\omega t}{2}-\frac{i}{\sqrt{2}} \cos \frac{\vartheta}{2} \sin \frac{\varphi+\omega t}{2}\right|^{2}= \\
& =\frac{1}{2}\left[\cos ^{2} \frac{\vartheta}{2} \cos ^{2} \frac{\varphi+\omega t}{2}+\sin ^{2} \frac{\vartheta}{2} \sin ^{2} \frac{\varphi+\omega t}{2}+\frac{1}{2} \sin \vartheta \sin (\varphi+\omega t)\right]+ \\
& +\frac{1}{2}\left[\sin ^{2} \frac{\vartheta}{2} \cos ^{2} \frac{\varphi+\omega t}{2}+\cos ^{2} \frac{\vartheta}{2} \sin ^{2} \frac{\varphi+\omega t}{2}+\frac{1}{2} \sin \vartheta \sin (\varphi+\omega t)\right]= \\
& =\frac{1}{2}\left[\cos ^{2} \frac{\varphi+\omega t}{2}\left(\cos ^{2} \frac{\vartheta}{2}+\sin ^{2} \frac{\vartheta}{2}\right)+\sin ^{2} \frac{\varphi+\omega t}{2}\left(\sin ^{2} \frac{\vartheta}{2}+\cos ^{2} \frac{\vartheta}{2}\right)+\sin \vartheta \sin (\varphi+\omega t)\right]= \\
& =\frac{1}{2}+\frac{1}{2} \sin \vartheta \sin (\varphi+\omega t)
\end{aligned}
$$

Analogously the probabilities of obtaining $-h$ in a measurement of the observable $L_{y}$ and $-\gamma h$ in a measurement of the observable $M_{y}$ are:

$$
\left|y \left(-\left.|\chi(t)\rangle_{\mathbf{L}}\right|^{2}=\frac{1}{2}-\frac{1}{2} \sin \vartheta \sin (\varphi+\omega t)\right.\right.
$$

so the matrices $L_{y}$ and $M_{y}$, which don't commute with $H$, have mean values:

$$
\begin{gathered}
{ }_{\mathbf{L}}\langle\chi(t)| L_{y}|\chi(t)\rangle_{\mathbf{L}}=\left(\begin{array}{ll}
\cos \frac{\vartheta}{2} e^{i(\varphi+\omega t) / 2} & \sin \frac{\vartheta}{2} e^{-i(\varphi+\omega t) / 2}
\end{array}\right) h\left(\begin{array}{rr}
0 & -i \\
i & 0
\end{array}\right)\binom{\cos \frac{\vartheta}{2} e^{-i(\varphi+\omega t) / 2}}{\sin \frac{\vartheta}{2} e^{i(\varphi+\omega t) / 2}}= \\
=h \sin \frac{\vartheta}{2} \cos \frac{\vartheta}{2}\left(-i e^{i(\varphi+\omega t)}+i e^{-i(\varphi+\omega t)}\right)=h \sin \vartheta \sin (\varphi+\omega t) \\
\mathbf{M}^{\langle }\langle\psi(t)| M_{y}|\psi(t)\rangle_{\mathbf{M}}=\gamma h \sin \vartheta \sin (\varphi+\omega t)
\end{gathered}
$$

which are clearly time-dependent. All the examined mean values are typical of Larmor precessions. More exactly $\left\langle L_{z}\right\rangle,\left\langle L_{x}\right\rangle$, and $\left\langle L_{y}\right\rangle$ behave like the components of a classical angular momentum having modulus $h$, while $\left\langle M_{z}\right\rangle,\left\langle M_{x}\right\rangle$, and $\left\langle M_{y}\right\rangle$ similarly behave but with a different modulus $\gamma \boldsymbol{h}$.

## The Nala's magic double dress

The $v$ particle, as we told above, plays a major role because it imposes on $\delta$ its favourite revolution which can be clockwise or counterclockwise (fig. 5) with the same probabilities (in the legend Nala has a magic double dress).


Figure 5
Thus we have an antiparallel "couple of couple" in a spectacular double cohort dance ${ }^{2}$, or that we call an entangled state, which needs to be studied in a four-dimensional state space, characterized by the orthonormal base $|++\rangle,|+-\rangle,|-+\rangle$ and $|--\rangle$. The entanglement is unavoidable, because it is the only way to perfectly neutralize the magnetic moments ${ }^{3}$ ! It involves a double angular momentum state DL and a double magnetic moment state DM. We know in advance that these states have the same eigenvectors and similar observables (which differ only for $h$ and $\gamma h$ ). Doing away with redundant calculations we can exclusively consider the double angular momentum state DL with the observables $D L_{z}, D L_{x}, D L_{y}, D L_{u}$, and the respective components $L_{1 z}$ and $L_{2 z}, L_{1 x}$ and $L_{2 x}, L_{1 y}$ and $L_{2 y}, L_{1 u}$ and $L_{2 u}$. All these observables and their entanglement compatible eigenvectors $|+-\rangle_{\mathbf{u}}$ and $|-+\rangle_{\mathbf{u}}$ are definable in the following way:

[^1]unfortunately a simple C.S.C.O. like $\left\{\mathbf{L}_{1}{ }^{2}, \mathbf{L}_{2}{ }^{2}, L_{1 z}, L_{2 z}\right\}$ with ${ }^{4}$ eigenvectors $|+-\rangle$ and $|-+\rangle$ is not suitable for the entanglement: we would have, for instance, $D L_{x}|+-\rangle \neq 0$ which is unacceptable; we need the C.S.C.O. $\left\{\mathbf{L}_{1}{ }^{2}, \mathbf{L}_{2}{ }^{2}, \mathbf{D L}{ }^{2}, D L_{z}\right\}$ for determining the right combination between $|+-\rangle$ and $|-+\rangle$.
The most direct way to determine the observable $\mathbf{D} \mathbf{L}^{2}$ is:
$$
\mathbf{D L}^{2}=D L_{x}^{2}+D L_{y}{ }^{2}+D L_{z}^{2}
$$
thus we obtain:
\[

\mathbf{D L}^{2}=4 h^{2}\left($$
\begin{array}{llll}
2 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 \\
0 & 1 & 1 & 0 \\
0 & 0 & 0 & 2
\end{array}
$$\right)
\]

[^2]It is easily calculable ${ }^{5}$ that this observable has one eigenvalue, three-fold degenerate $\left(8 h^{2}\right)$, with the triplet of eigenvectors $|++\rangle,|--\rangle$ and $1 / \sqrt{2}(|+-\rangle+|-+\rangle)$, and one null eigenvalue, nondegenerate, with the singlet eigenvector $1 / \sqrt{2}(|+-\rangle-|-+\rangle)$ which is representative of the entangled state $|\boldsymbol{\varepsilon}\rangle$.
In a measurement of the generic observable $D L_{u}$ (therefore for any observable) we have the same probabilities to obtain the states $|+-\rangle_{\mathbf{u}}\left(+h\right.$ for $L_{1 u}$ and $-h$ for $\left.L_{2 u}\right)$ and $|-+\rangle_{\mathbf{u}}\left(-h\right.$ for $L_{I u}$ and + 4 for $L_{2 u}$ ):

$$
\left.\begin{array}{l}
\left.\left.\right|_{\mathbf{u}}\langle+-\mid \boldsymbol{\varepsilon}\rangle\right|^{2}=\left\lvert\,\left(-\sin \frac{\vartheta}{2} \cos \frac{\vartheta}{2} e^{i \varphi} \quad \cos ^{2} \frac{\vartheta}{2}\right.\right.
\end{array}-\sin ^{2} \frac{\vartheta}{2} \quad \sin \frac{\vartheta}{2} \cos \frac{\vartheta}{2} e^{-i \varphi}\right)\left.\left(\begin{array}{c}
0 \\
\frac{1}{\sqrt{2}} \\
-\frac{1}{\sqrt{2}} \\
0
\end{array}\right)\right|^{2}=\left(\frac{1}{\sqrt{2}}\right)^{2}=\frac{1}{2} .
$$

We can see that the normalized projections of $|\boldsymbol{\varepsilon}\rangle$ onto $|+-\rangle_{\mathbf{u}}$ or $|-+\rangle_{\mathbf{u}}$ coincide with the same eigenvectors, while the mean value is null:

$$
\langle\boldsymbol{\varepsilon}| D L_{u}|\boldsymbol{\varepsilon}\rangle=\left(\begin{array}{llll}
0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0
\end{array}\right) h\left(\begin{array}{cccc}
2 \cos \vartheta & \sin \vartheta e^{-i \varphi} & \sin \vartheta e^{-i \varphi} & 0 \\
\sin \vartheta e^{i \varphi} & 0 & 0 & \sin \vartheta e^{-i \varphi} \\
\sin \vartheta e^{i \varphi} & 0 & 0 & \sin \vartheta e^{-i \varphi} \\
0 & \sin \vartheta e^{i \varphi} & \sin \vartheta e^{i \varphi} & -2 \cos \vartheta
\end{array}\right)\left(\begin{array}{c}
0 \\
\frac{1}{\sqrt{2}} \\
-\frac{1}{\sqrt{2}} \\
0
\end{array}\right)=\left(\begin{array}{l}
0 \\
0 \\
0 \\
0
\end{array}\right)
$$

It is easy to deduce that the normalized projection of $|\boldsymbol{\varepsilon}\rangle$ onto any other eigenstate, chosen among
 $\left\langle D L_{x}\right\rangle$ and $\left\langle D L_{y}\right\rangle$ are null.
As we told above the double magnetic moment state DM has the same eigenvectors and all its mean values $\left\langle D M_{z}\right\rangle,\left\langle D M_{x}\right\rangle,\left\langle D M_{y}\right\rangle$ and $\left\langle D M_{u}\right\rangle$ are obviously null.

A complicated Stern-Gerlach apparatus



Figure 6

[^3]The experiment could make us see the particular deflection of a beam of light (consisting of paramagnetic photons) in an inhomogeneous magnetic field (fig 6).
Photons coming from a light bulb $L$ get in a focussing device $F$ which selects the photons with velocity parallel to the $y$ axis; the collimated beam crosses an electromagnet $E$, remaining divided in two beams (with opposite deflections) by a strong magnetic gradient, and, after being analysed by two polaroids $P_{1}$ and $P_{2}$, lights up the wall $A$ in two distant spots $N_{l}$ and $N_{2}$. The magnetic field $\mathbf{B}$, with $y z$ plane of symmetry, has no component along $y$; it strongly varies with $z$ and, being $\boldsymbol{\nabla} . \mathbf{B}=0$, also varies with $x$.
From the potential energy $W=-2 \mathscr{M}_{v} . \mathbf{B}$ it is possible to deduce the resultant of the forces exerted on the photon:

$$
\mathbf{F}=\nabla\left(2 \mathscr{M}_{v}, \mathbf{B}\right)=\nabla\left(\mathscr{M}_{\mathrm{tot}} . \mathbf{B}\right)
$$

neglecting $\mathscr{T}_{\mathcal{H}_{\text {totx }}}$ because of the high frequency of oscillation due to the Larmor precession ( $\mathscr{M}_{\text {toty }}$ gives no worries since $B_{y}=0$ ), we have:

$$
\mathbf{F}=\boldsymbol{\nabla}\left(\mathscr{M}_{\text {totz }} B_{z}\right)=\mathscr{M}_{\text {totz }} \boldsymbol{\nabla} B_{z}
$$

but $\partial B_{z} / \partial y=0$, because $\mathbf{B}$ is independent of $y$, and $\partial B_{z} / \partial x=0$ at any point belonging to the plane of symmetry $y z$. The force acting on the photon is therefore parallel to the $z$ axis and proportional to $\mathscr{\mathscr { M }}_{\text {totz }}$ and $\partial B_{z} / \partial z$; the same force depends on the sign of $\mathscr{M}_{\mathcal{H}_{\text {totz }}}$ (therefore on the direction of $\mathscr{M}_{\text {tot }}$ ), because $\partial B_{z} \partial z$ is only influenced by the geometry of the magnet poles. The structure of the photon is more amazing than we would have expected: the polarization can affect both the angular momentum $\mathbf{L}_{\text {tot }}$ of $\delta$ and $v$ particles and their magnetic moment $\mathscr{\mathscr { M }}_{\text {tot }}$, that's to say even the nullresultant spins! Thus the apparatus disposed like in figure 6 prepares the states:

$$
\begin{gathered}
|\chi\rangle_{\mathbf{L}}=|+\rangle \quad \text { and } \quad|\psi\rangle_{\mathbf{M}}=|+\rangle \text { plus } \\
|\chi\rangle_{\mathbf{L}}=|-\rangle \quad \text { and } \quad|\psi\rangle_{\mathbf{M}}=|-\rangle
\end{gathered}
$$

eigenvectors of $L_{z}$ and $M_{z}$; rotating the magnet through the angle $+\pi / 2$ on the $y$ axis we have:

$$
\begin{gathered}
|\chi\rangle_{\mathbf{L}}=\frac{1}{\sqrt{2}}(|+\rangle+|-\rangle) \quad \text { and } \quad|\psi\rangle_{\mathbf{M}}=\frac{1}{\sqrt{2}}(|+\rangle+|-\rangle) \quad \text { plus } \\
|\chi\rangle_{\mathbf{L}}=\frac{1}{\sqrt{2}}(|+\rangle-|-\rangle) \quad \text { and } \quad|\psi\rangle_{\mathbf{M}}=\frac{1}{\sqrt{2}}(|+\rangle-|-\rangle),
\end{gathered}
$$

eigenvectors of $L_{x}$ and $M_{x}$. For a generic angle $\vartheta(<\pi)$ of the unit vector $\mathbf{u}$ lying on $x z$ plane $(\varphi=0)$, the observable $L_{u}$ becomes:

$$
L_{u}=h\left(\begin{array}{rr}
\cos \vartheta & \sin \vartheta \\
\sin \vartheta & -\cos \vartheta
\end{array}\right)
$$

and the polarized states are:

$$
\begin{gathered}
|\chi\rangle_{\mathbf{L}}=\cos \frac{\vartheta}{2}|+\rangle+\sin \frac{\vartheta}{2}|-\rangle \quad \text { and } \quad|\psi\rangle_{\mathbf{M}}=\cos \frac{\vartheta}{2}|+\rangle+\sin \frac{\vartheta}{2}|-\rangle \quad \text { plus } \\
|\chi\rangle_{\mathbf{L}}=-\sin \frac{\vartheta}{2}|+\rangle+\cos \frac{\vartheta}{2}|-\rangle \quad \text { and } \quad|\psi\rangle_{\mathbf{M}}=-\sin \frac{\vartheta}{2}|+\rangle+\cos \frac{\vartheta}{2}|-\rangle
\end{gathered}
$$

eigenvectors of $L_{u}$ and $M_{u}$.
What's the problem then? As the time needed to cross the gap of the electromagnet is of the order of $10^{-10} \mathrm{~s}$, the expected deflection angle is too little to be noticed without resorting to expedients. A fundamental work, carried out by Karpa and Weitz ${ }^{6}$, shows us that light passing through a rubidium gas cell $\left(\mathrm{v}_{\mathrm{g}} \cong 300 \mathrm{~m} / \mathrm{s}\right)$, under electromagnetically induced transparency, is deflected by a small magnetic gradient $\left(10^{-4} \mathrm{~T} / \mathrm{m}\right)$ through an angle of $10^{-4} \mathrm{rad}$. At vacuum light velocity conditions the deflection is dramatically reduced of 12 orders of magnitude ( $10^{-16} \mathrm{rad}$ ); even if we think about a stronger magnetic gradient $\left(10^{-1} \mathrm{~T} / \mathrm{m}\right)$ we can reach a maximum deflection of $10^{-13}$ rad: buffer gas cells or other optical tricks are absolutely necessary!
The enormous difficulty of pointing out this trifling deflection made us resolutely (but wrongly!) state: "No magnetic moment exists for photons in vacuum!". On the contrary photons, in any condition, have their intrinsic magnetic moment which exactly coincides with the Bohr magneton $\mu_{\mathrm{B}}{ }^{7}$. An important confirmation is, for instance, the paramagnetism of the hydrogen atom; the relation:

$$
\mathbf{M}=\frac{q}{2 m_{e}} \mathbf{L}
$$

between the operators of magnetic moment $\mathbf{M}$ and angular momentum $\mathbf{L}$ has the eigenvalues of any component of $\mathbf{M}$ in the form:

$$
\lambda=\left(\frac{q}{2 m_{e}}\right)(m h)=m \mu_{\mathrm{B}} \quad \text { with } \mu_{\mathrm{B}}=\frac{q h}{2 m_{e}}
$$

where $m$ is an integer, and the Bhor magneton $\mu_{\mathrm{B}}$ the integer contribution made by one photon! It is not difficult to deduce that $\boldsymbol{v}, \boldsymbol{\delta}$ and the electron have the same ratio between electric charge and mass: that's why they can vibrate in syntony! Other confirmations are all the atoms with paramagnetic behaviour.

## The inebriated state

Applying the Nala-Damayanti model to the visible light something interesting emerges: the orbits intersect a rainbow mandorla (fig. 7) which had great importance among ancient civilizations, till the Gothic one: why?


Figure 7

[^4]It is clear that the maximum probability of finding the $v$ and $\delta$ particles is inside the mandorla; the large radii of curvature of these orbital branches make us think about a velocity slowdown (because the total angular momentum $\mathbf{L}_{\text {tot }}$ is a constant of the motion). When the two particles are nearing each other they have an enlargement of their diameter as far as the interpenetration (the legend is extremely precise in describing Nala and Damayanti in continuous intimacy!); this condition minimizes the effect of the electrical charges and the binding strength between $\boldsymbol{v}$ and $\boldsymbol{\delta}$ particles is reduced: it is like if they got in an inebriated state and had a long, vibrating embrace, full of love for each other; that being so we must stop thinking about particles like spheres because they are more similar to elasticated disks. We met first with a similar problem in 2010 when the proton radius of muonic hydrogen (proton with negative muon) came up with a significantly lower value than that of canonical hydrogen (proton with electron) showing an elastic behaviour of the proton ${ }^{8}$.
Unfortunately, our QED is not enough to explain this phenomenon: the cloud-like wave function of the two particles is spread out over the " S " orbital and the maximum probability of finding the particles is simply at the centre of a sphere: we are far enough from the real conditions.


Figure 8
The orbits of $\boldsymbol{v}$ and $\boldsymbol{\delta}$ particles describe in the space a delicious apple (fig 8 ) and the maximum probability of finding the two particles is in its core; but as the pips cannot stay at the centre of the apple, so $\boldsymbol{v}$ or $\delta$ never stay at the centre of the " S " orbital! This apple-like structure of the photon clearly emerges from another important ancient sacred text ${ }^{9}$.

## Conclusions

The only concern of this paper has been confining ourselves to the internal degree of freedom of the photon. Taking a wider look at the wonderful couple $v-\delta$ could be traumatic: many myths could explode. Truth shall fortunately make them explode!

[^5]
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[^0]:    ${ }^{1}$ Mahabharata, III (Vana Parva), 53,1-79,5.

[^1]:    ${ }^{2}$ See note 8.
    ${ }^{3}$ No mechanical perturbation is generated by two antiparallel magnetic moments.

[^2]:    ${ }^{4} \mathbf{L}_{1}{ }^{2}=\mathrm{L}_{1 \mathrm{x}}{ }^{2}+\mathrm{L}_{1 \mathrm{y}}{ }^{2}+\mathrm{L}_{1 z}{ }^{2}=3 h^{2} I$ and $\mathbf{L}_{2}{ }^{2}=\mathrm{L}_{2 \mathrm{x}}{ }^{2}+\mathrm{L}_{2 y}{ }^{2}+\mathrm{L}_{2 z}{ }^{2}=3 h^{2} I$.

[^3]:    ${ }^{5} \operatorname{Det}\left[\mathbf{D L}^{2}-\lambda I\right]=\left(8 h^{2}-\lambda\right)\left(8 h^{2}-\lambda\right)\left[\left(4 h^{2}-\lambda\right)^{2}-16 h^{4}\right]=\left(8 h^{2}-\lambda\right)\left(8 h^{2}-\lambda\right) \lambda\left(\lambda-8 h^{2}\right)=0$.

[^4]:    6 "A Stern-Gerlach experiment for slow light". (See bibliography).
    ${ }^{7}$ Many scientists hurriedly gave the dark state polaritons a magnetic moment $\mu_{\mathrm{B}}$, which is an exclusive of the photon.

[^5]:    8 "The size of the proton". (See bibliography).
    ${ }^{9}$ Song of Songs $(2,3 ; 2,5 ; 8,5)$; in the Hebrew culture the couple of this sacred book symbolizes the photon, while the entanglement is a double cohort dance ( 7,1 ). All the ancient civilizations symbolized the photon by myths or legends about indivisible couples, like Nala and Damayanti: the couple of the Love Songs with Mehi like Eros (Egypt), Eros and Psyche (Greece, Rome), Laila and Majnun (Arabs), the Legend of flutes (Redskin Indians), the First Human Couple from Titicaca (Andeans), ...

